UDC 532.517:537.84

ALIGNMENT IN FREE DECAYING MHD TURBULENCE

T. Antonov[1], **P. Frick**[1], **D. Sokoloff**[2]

Using a shell model of turbulence, we have investigated a long-time evolution of magnetic and velocity fields in the MHD turbulence with various types of initial states. For each kind of these states, 24 realizations of the process with random noise in the initial states were modeled simultaneously with the help of a parallel computer cluster at the Research Computing Center of Moscow State University. For most realizations, a coherent state with high alignment between the magnetic and velocity fields has been developed. In the case of establishment of the coherent state, the dissipation rate of energy is found to be substantially reduced. At the same time, there exist few realizations that behave in another manner: they are characterized by a lower level of cross-helicity or an essentially faster energy decay.

1. Introduction. Turbulent flows in an electrically conducting fluid can generate a random, chaotic magnetic field provided that the magnetic Reynolds number Rm is large enough. This process is known as a small scale dynamo and is involved (together with a large scale dynamo supported by helicity and differential rotation) in evolution of cosmic magnetic fields. A magnetic field generated by a small scale dynamo grows from a seed field to equipartition at a time scale of order l/v (where l/v is the turnover time, l is the basic scale of turbulence, and v is the corresponding velocity) and remains statistically stable at the time scale much larger than l/v.

This equilibrium MHD (magnetohydrodynamic) state can be described in terms of its kinetic and magnetic spectra, spectral flux, etc. Three inviscid integrals of motion provide a large variety of scaling properties for MHD turbulence. The idea of a constant spectral energy flux leads to the Kolmogorov spectral index "$-5/3$" [1]. The concept of the Alfvénic wave turbulence results in the Kraichnan–Iroshnikov spectrum "$-3/2$" [2, 3]. The third possibility was suggested by Dobrovolny et al. [4] and developed by Pouquet et al. [5]. This scenario suggests high correlation of the velocity and magnetic field configurations, which locks the cascade. Such a state subjected only to molecular dissipation generates almost parallel velocity $\mathbf{u}$ and magnetic $\mathbf{B}$ fields and is referred as alignment.

The alignment can be quantified by a correlation coefficient $C = \dfrac{H_C}{(E_B + E_U)}$, where $H_C = \int \mathbf{uB}\, d\mathbf{r}$ is the cross-helicity, E_U and E_B are the kinetic and magnetic energies, respectively. In contrast to the mean-field dynamo, the small scale dynamo is not associated with any kind of helicity and can operate in mirror-symmetric turbulence. Thus, in a flow with initially weak cross-helicity, the alignment is expected to take place only in a late stage of the evolution.

The appearance of the alignment concept was motivated in part by the presence of strong $\mathbf{u} - \mathbf{b}$ correlations in solar wind observations and by some indications of growth of the correlation coefficient C obtained from direct numerical simulations (DNS) [5]. However, the possibilities of DNS are obviously restricted, and it seems more attractive to use much less time-consuming models for long-time simulations. For this purpose, we used a shell model of MHD turbulence introduced in [6]. This model has the advantage of containing all MHD integrals of motion and reproducing fine details of the small-scale dynamo.

The appearance of alignment in a stationary forced MHD turbulence has been observed in the shell model numerical simulations considered in [7]. However, the long-time behavior essentially depends on forcing. In particular, a random force can result in a chaotic sequence of epochs with $C = 1$ and $C = -1$. These reversals of alignment in the forced shell model can be attributed to the specific features of forcing. The free decay model does not result in such dramatic behavior. In particular, the total energy decays in time without pronounced alignment reversals have been observed [7]. However, we demonstrate below that the alignment is typical for the free decay model as well as for the forced one.

2. Shell model. The basic idea of any shell model of fully developed turbulence is to retain only one real or complex mode (in our case, the complex variables U_n and B_n correspond to the velocity and magnetic fields) as a representative of all modes in the shell with the wave number $k_n < |\mathbf{k}| < k_{n+1}$, $k_n = 2^n$, and to introduce a system of ordinary differential equations that mimics the original nonlinear partial differential equations (for an introduction to shell models see, for example, [8]).

[1] Institute of Continuous Media Mechanics, Korolyov 1, 614061 Perm, Russian Federation
[2] Research Computing Center, Moscow State University, 119899 Moscow, Russian Federation, e-mail: sokoloff@dds.srcc.msu.su

In this paper, we use the MHD-shell model introduced in [6] and rewritten here for 3D turbulence as

$$\left(d_t + \mathrm{Re}^{-1} k_n^2\right) U_n = ik_n \Big\{(U_{n+1}^* U_{n+2}^* - B_{n+1}^* B_{n+2}^*) - \frac{1}{4}(U_{n-1}^* U_{n+1}^* - B_{n-1}^* B_{n+1}^*) + \frac{1}{8}(U_{n-2}^* U_{n-1}^* - B_{n-2}^* B_{n-1}^*)\Big\} \quad (1)$$

$$\left(d_t + \mathrm{Rm}^{-1} k_n^2\right) B_n = \frac{ik_n}{6}\Big\{(U_{n+1}^* B_{n+2}^* - B_{n+1}^* U_{n+2}^*) + (U_{n-1}^* B_{n+1}^* - B_{n-1}^* U_{n+1}^*) + (U_{n-2}^* B_{n-1}^* - B_{n-2}^* U_{n-1}^*)\Big\} \quad (2)$$

In these equations, Re is the Reynolds number and Rm is the magnetic Reynolds number. If $B_n = 0$, one obtains the so-called GOY shell model [8 – 10] widely used for the Navier–Stokes equations.

In the limit Re, Rm $\to \infty$, equations (1) and (2) retain three quadratic quantities corresponding to the three quadratic invariants of the inviscid MHD flows: the total energy $E = E_V + E_B$, the cross helicity H_C, and the magnetic helicity H_B [6]. In terms of the shell model, they are expressed as

$$E_U = \sum_n |U_n|^2, \quad E_B = \sum_n |B_n|^2, \quad H_C = \sum_n (U_n B_n^* + U_n^* B_n), \quad H_B = \sum_n (-1)^n k_n^{-1} |B_n|^2$$

Note that the nonlinear terms of (1) and (2) identically vanish for $U_n = \pm B_n$ (the Alfvénic fixed points) and the spectral energy flux is blocked as well.

3. Numerical implementation. The numerical results have been obtained on a parallel computing cluster at the Research Computing Center of Moscow State University. The cluster contains 24 Intel processors PIII-550. Equations (1) and (2) with Re = Rm = 10^{-9} were integrated by a fourth-order Runge-Kutta method with the constant time step $2 \cdot 10^{-6}$.

The problem under consideration is suited ideally for modeling on parallel computers. This allows us to reproduce the evolution of many individual systems for various initial states. Thus, we generated 24 random initial sets of U_n and B_n with given integral characteristics and spectral distribution. The data exchange between the processors being minimal during the simulations is used for statistical studies only. The advantage of the code is that even a failure of a processor does not cause serious complications: only one of 24 realizations is lost in the event of the processor failure.

The simulation up to the turnover time $t = 9600$ has taken about 50 hours of processing time.

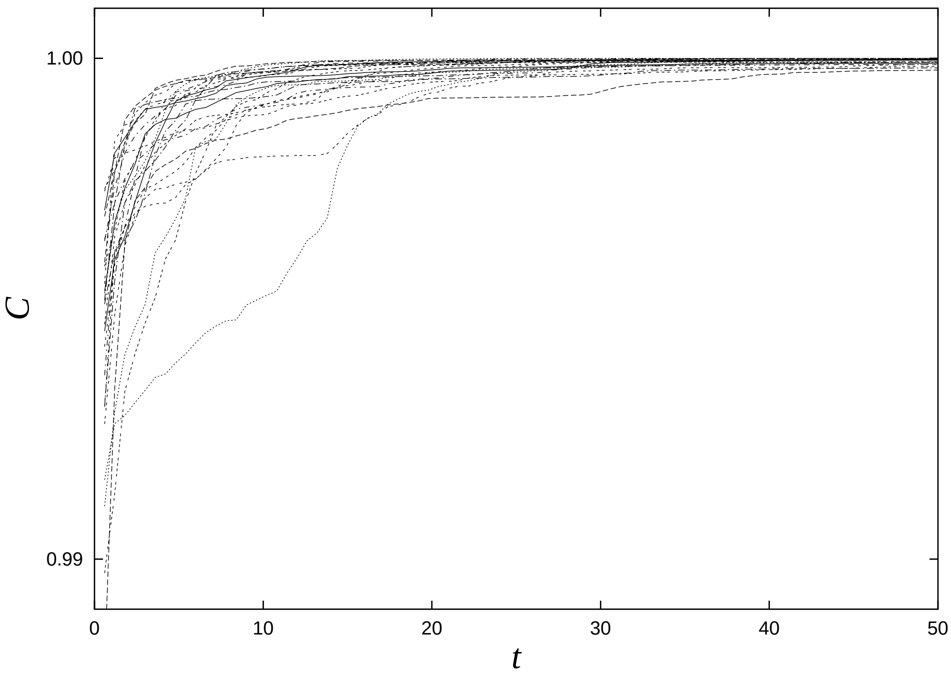

Fig. 1. Time evolution of the correlation coefficient with initially prescribed equipartition ($E_B \approx E_U$)

4. Results. Our first numerical experiment concerns the turbulence with initially prescribed equipartition ($E_B \approx E_U$) and the high level of correlation ($C \approx 0.9$). Figure 1, where the time evolution of the correlation coefficient is shown, demonstrates that in few dozens of turnover times all solutions are attracted by a state with strong alignment ($C \to 1$).

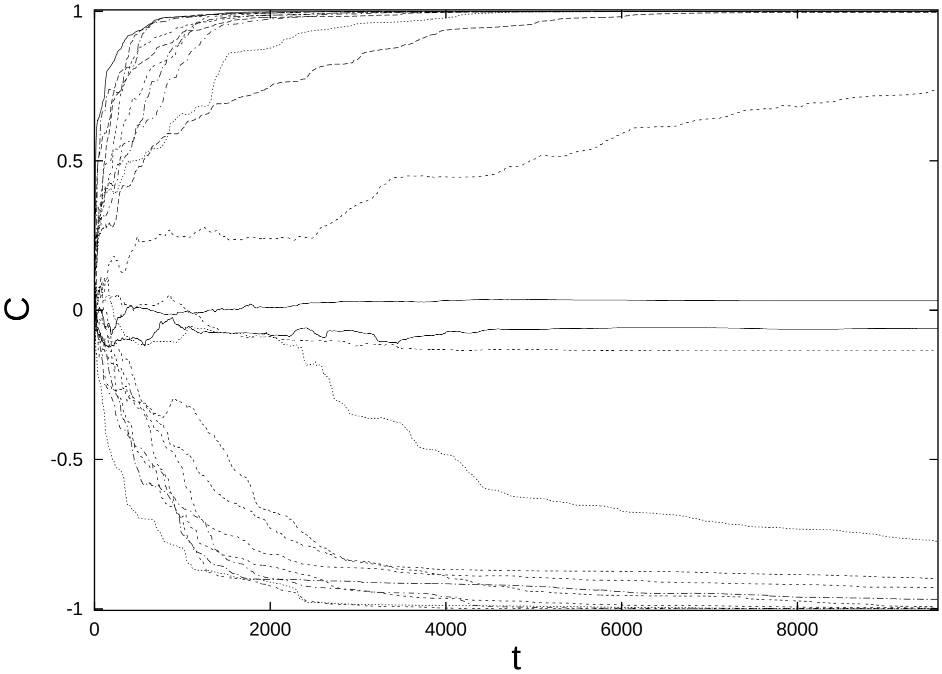

Fig. 2. Time evolution of the correlation coefficient C with initially weak magnetic fields ($E_B \ll E_U$)

Our second numerical experiment concerns the free-decaying turbulent dynamo problem in which a well developed kinetic energy spectrum with a weak magnetic energy ($E_B \ll E_U$) is assumed to be an initial state. This approach can be considered as that of a dynamo problem, because the time of energy decay is much longer than the characteristic time of magnetic field growth. The time about few dozens of turnover times is found to be sufficient for reaching the equipartition of the magnetic and kinetic energies [6].

The simulations performed for time scales of thousands turnover times show that the set of solutions with similar initial conditions displays substantially different types of behavior. Figure 2 shows that in major cases the correlation coefficient C is attracted by one of the two extreme values $C = \pm 1$. A comparison of evolution tracks of C with the trajectories of a Brawnian particle reveals an enlargement of dispersion. Although the number of trajectories with significant C increases with time, the population of trajectories for which C remains small is still visible.

A sign of alignment is determined by the initial condition. If a given trajectory has a low alignment for a long time, the sign of C can reverse. However, we do not observe the sign reversal for a state with high alignment.

Note that the state with strong alignment corresponds to the exact solution $U_n = B_n$ of (1) and (2). This solution decays with time, but its decay rate is much lower than that in the case of low correlation. The cause of this difference is traced to the fact that the spectral flux of the coherent state vanishes, so that the magnetic and kinetic energies are subjected to molecular dissipation only. This two corresponding modes of the spectral energy distribution are shown in Figure 3. The white circles indicate the kinetic energy E_U and the black circles indicate the magnetic energy E_B. One spectrum (the large circles) corresponds to the noncorrelated state characterized by a well pronounced inertial range with a spectral index close to the Kolmogorov index "$-5/3$". The other spectrum (the small circles) belongs to a realization with a strong alignment. In this case, the main part of energy is concentrated in the three largest scales. A relatively short range of scales with a spectral index close to the Kraichnan–Iroshnikov index "$-3/2$" precedes the dissipative range. It should be emphasized that the dissipative scale in the last case is much larger than in the previous one, despite the fact that the total energy of the aligned state is two orders of magnitude larger than that of the noncorrelated state.

In Figure 4, the time dependence of total energy is represented in log-log coordinates for all realizations. This kind of representation is interesting from the viewpoint of power-law energy decay. It is clear that no universal time scaling is available. Most part of time tracks displays power-law behavior at the intermediate stage of evolution. This stage seems to be shorter for states with rapidly developed alignment. The slop of these tracks tends to "-0.5" (this slop is shown in Figure 4 by the thick solid line). For low-correlated states, this stage is longer and the slop tends to "-1". Both asymptotic slopes were observed by Biscamp and Müller [11] in DNS of decaying MHD turbulence. The power-low decay turns to a state with practically stable energy (the horizontal tails in Figure 4): this indicates the establishment of alignment.

5. Discussion. The free evolution of fully developed turbulence of an electrically conducting fluid with an initially

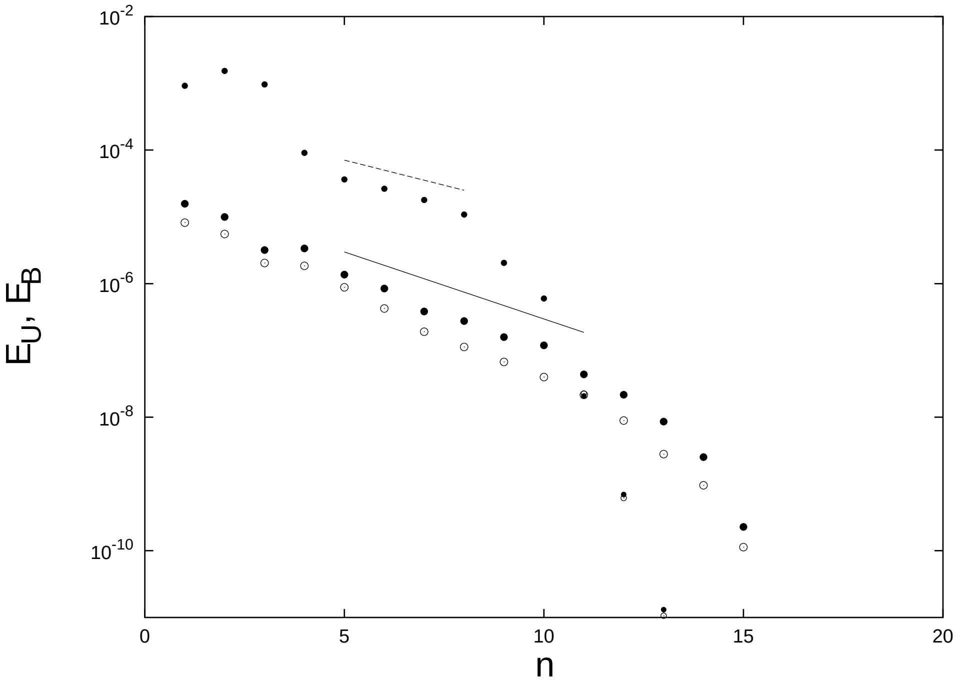

Fig. 3. The spectral energy distributions for a noncorrelated state (the large circles) and for a state with strong alignment (the small circles). The solid and dashed lines indicate the Kolmogorov "$-5/3$" and Kraichnan–Iroshnikov "$-3/2$" spectra, respectively

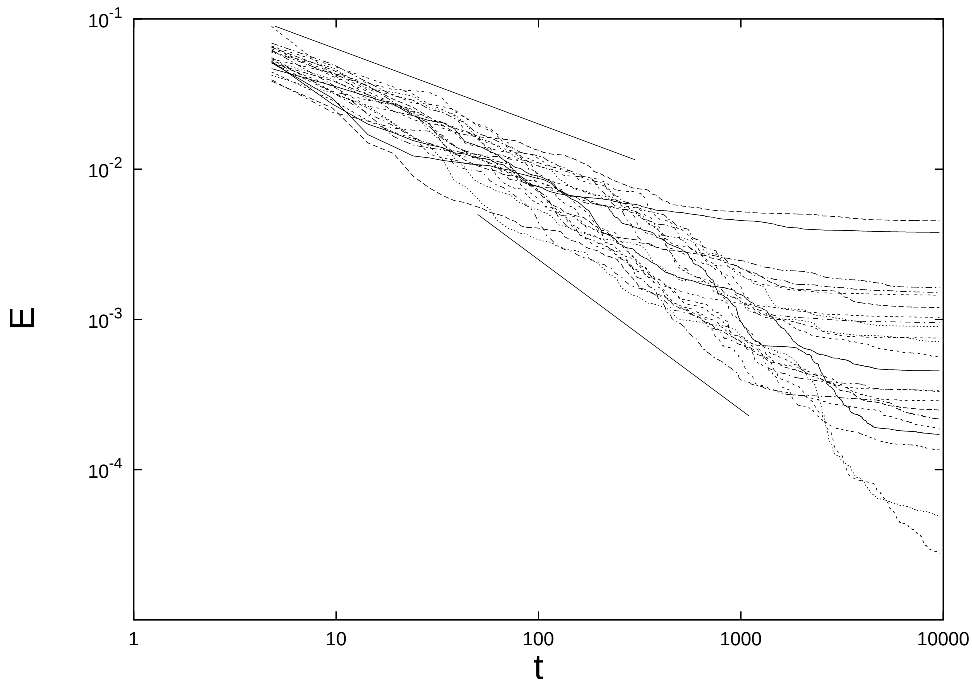

Fig. 4. Time evolution of total energy

weak magnetic field passes through a sequence of diverse stages. Starting from the kinematic growth of the magnetic field, it tends to the state characterized by the equipartition of the magnetic and kinetic energies with the developed spectral flux toward the small scales and moderate level of cross-correlation. A slow growth of correlation during this intermediate stage leads to a final stage characterized by a strong alignment of the velocity and magnetic fields, by the absence of energy cascade, and by steep spectra.

The above investigation of long-time evolution of a family of free decaying solutions for the MHD shell model demonstrates a large variety of individual behavior. For a real MHD turbulence, this fact can be interpreted as a strong space inhomogeneity, i.e., as intermittence. For this reason, the averaged characteristics of the MHD turbulence at the late stage of evolution are unable to give an adequate description of the turbulent state. The physical nature of strong intermittence in the MHD turbulence at the final stage of its evolution is the alignment of the magnetic and velocity fields.

This work was realized at the Research Computing Center, Moscow State University thanks to kind suggestion and fruitful help of A. V. Tikhonravov, Vl. V. Voevodin, and A. A. Kulikovsky. The work was partially supported by the Russian Foundation for Basic Research (99–01–00362).

REFERENCES

1. G.S. Golitsin, “Principle of the fastest response in hydrodynamics, geophysics, and astrophysics”, *Dokl. Ross. Akad. Nauk*, **356**, 3: 321–324, 1997.

2. R.H. Kraichnan, “Inertial-range spectrum of hydromagnetic turbulence”, *Phys. Fluids*, 8: 1385–1387, 1965.

3. R.S. Iroshnikov, “Turbulence of a conducting fluid in a strong magnetic field”, *Sov. Astron.*, 40: 742–750, 1963.

4. M. Dobrowolny, A. Mangeney, and P.-L. Veltri, “Fully developed anisotropic hydromagnetic turbulence in interplanetary space”, *Phys. Rev. Letters*, **45**, 2: 144–147, 1980.

5. A. Pouquet, M. Menneguzzi, and U. Frisch, “Growth of correlations in magnetohydrodynamic turbulence”, *Phys. Rew. A*, **33**, 6: 4266–4276, 1986.

6. P. Frick and D. Sokoloff, “Cascade and dynamo action in a shell model of magnetohydrodynamic turbulence” *Phys. Rev. E*, **57**, 4: 4155–4164, 1998.

7. G. Boffetta, P. Frick, P. Giuliani, S. Lozhkin, and D. Sokoloff, submitted to Europhys. Letters.

8. T. Bohr, J. Jensen, G. Paladin, and A. Vulpiani, *Dynamical Systems Approach to Turbulence*, Cambrige, 1998.

9. L. Biferale, A. Lambert, R. Lima, and G. Paladin, “Transition to chaos in a shell model of turbulence” *Physica D*, **80**, 1–2: 105, 1995.

10. P. Frick, B. Dubrulle, and A. Babiano, “Scaling properties of a class of shell models”, *Phys. Rev. E*, **51**, 6: 5582–5592, 1995.

11. D. Biskamp and W.-C. Müller, “Decay laws for three-dimensional magnetohydrodynamic turbulence”, *Phys. Rev. Letters*, **33**, 11: 2195–2198, 1999.

11 May 2000